MATH 4010 Functional Analysis, Homework 1. Deadline: 10 Feb 2017

1. Let A be a subset of a metric space X.

   (a) Show that if X is complete, then A is complete if and only if A is closed in X.

   (b) Show that if A is complete, then A is closed in X.

2. A metric d on X is said to be non-archimedean if it satisfies the strong triangle inequality, that is, $d(x, y) \leq \max(d(x, z), d(z, y))$ for all x, y and $z \in X$. Show that if d is a non-archimedean metric on X, then for every closed ball $\overline{B}(a, r) := \{x \in X : d(a, x) \leq r\}$ is an open set in X.